Tricritical behavior in the Sherrington-Kirkpatrick spin glass under a bimodal random field

E. Nogueira, Jr.,[1,2] F. D. Nobre,[1] F. A. da Costa,[1] and S. Coutinho[1,3]
[1]*Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, Campus Universitário, Caixa Postal 1641 59072-970 Natal, Rio Grande do Norte, Brazil*
[2]*Instituto de Física, Universidade Federal da Bahia, Campus Universitário de Ondina, 40210-340 Salvador, Bahia, Brazil*
[3]*Laboratório de Física Teórica e Computacional, Universidade Federal de Pernambuco, Cidade Universitária, 50670-901 Recife, Pernambuco, Brazil*
(Received 6 November 1997)

The infinite-range-interaction Ising spin glass, in the presence of an external random field, is investigated through the replica method. At each site, the field follows a bimodal distribution, assuming the values $\pm h_0$. Within the replica-symmetry approximation, the phase diagram is obtained for different values of h_0. The border of the ferromagnetic phase displays interesting behavior, depending on the value of h_0, with two threshold values ($h_0^{(1)}$ and $h_0^{(2)}$): (i) a continuous line, for $h_0 < h_0^{(1)}$; (ii) two pieces, one continuous (high temperatures) and another of the first-order type (low temperatures), connected at a tricritical point, for $h_0 > h_0^{(2)}$; and (iii) two continuous pieces (high and low temperatures) and a first-order part in between, with two tricritical points, for $h_0^{(1)} \leqslant h_0 \leqslant h_0^{(2)}$. The stability of the replica-symmetric solution is analyzed. It is shown that the higher-temperature tricritical point is always in a stable region of the phase diagram, whereas the lower-temperature one is, most of the time, inside the unstable region. Along the first-order critical line, a small gap is found between the borders associated with the instabilities of the replica-symmetric solution from either side of the phase-coexistence region, i.e., these instability lines do not meet at the ferromagnetic frontier, as usually happens in the case of second-order phase transitions. [S1063-651X(98)07705-8]

PACS number(s): 05.50.+q, 64.60.−i, 75.10.Nr, 75.50.Lk

I. INTRODUCTION

Disordered magnets [1] have become one of the most exciting areas in magnetism from both theoretical and experimental points of view. Among many interesting systems, two of them may be singled out as sources of remarkable controversies, namely, spin glasses [2,3] and the ferromagnet in the presence of a random field [3,4].

Most of the spin-glass theory has been concentrated at the mean-field level, based on infinite-range-interaction models, whose prototype is formulated for the Ising spin glass (ISG), the so-called Sherrington-Kirkpatrick (SK) model [5]. The solution of the SK model presents unusual properties, such as the Almeida-Thouless (AT) line [6], which, in the presence of an external uniform magnetic field, separates a high-temperature region where the spin-glass order parameter is unique from a low-temperature one, defined in terms of an infinite number of order parameters, i.e., an order-parameter function [7]. Within the replica-method [2] terminology, the phase characterized by a single parameter is said to follow replica symmetry, whereas at low temperatures the instability of replica symmetry is corrected by the introduction of Parisi's order-parameter function, a procedure that is usually denominated replica symmetry breaking (RSB). Such an order-parameter function is directly related to a multiplicity of equilibrium states at low temperatures, which are organized in a hierarchical structure, defining an ultrametric space [8]. Concerning short-range-interaction systems, an early controversy referred to the lower critical dimension d_l of the ISG. This question is by now completely settled, with numerical simulations [9], high-temperature series expansions [10], and renormalization-group methods [11,12] all agreeing that $2 < d_l < 3$ in such a way that there is a phase transition in three dimensions, but not in two. Whether RSB survives in real short-range spin glasses has turned into a polemic, not yet resolved [13]. The rival theory is the droplet model [14], based on domain-wall renormalization-group arguments for spin glasses [12,15]. Contrary to RSB, the droplet model describes the low-temperature phase of any short-range *finite-dimensional* spin glass in terms of a single thermodynamic state (together, of course, with its corresponding time reverse). The droplet model goes through difficulties as the dimensionality increases, since one expects the existence of a finite upper critical dimension (believed to be 6 for the ISG [16]) above which the mean-field picture should become valid. It is very difficult to carry out numerical simulations in dimension 3, which is presumably close to the lower critical dimension, due to thermalization difficulties; as a consequence, no conclusive results concerning this controversy in three-dimensional systems are available. However, in four dimensions the critical temperature is much higher, making thermalization easier; many works claim to have observed some mean-field features in this case [17].

The random-field Ising model (RFIM), as introduced by Imry and Ma [18], has concentrated much attention on the revelation of its physical realization as a diluted Ising antiferromagnet in the presence of a uniform magnetic field [19] and that the static critical properties in these two systems may be the same [20]. Simple physical arguments due to Imry and Ma suggested that the lower critical dimension of the RFIM, above which there exists a stable ferromagnetic state at low temperatures, should be $d_l = 2$; although that point remained controversial for some time, rigorous results [21] showed that the assertion is indeed true. According to its mean-field theory, the nature of the phase transition depends on the distribution associated with the magnetic field. In the Gaussian case, the phase transition is always continuous [22], whereas for a symmetric bimodal distribution (for which the field assumes the values $\pm h_0$ with equal probabilities), the phase transition is continuous for h_0 small and high temperatures, becoming first order for sufficiently large values of h_0 and low temperatures [23]. In the latter case, the critical frontier presents a tricritical point connecting the continuous and first-order pieces. Analogous to what happens for spin glasses, the free-energy landscape at low temperatures may be complicated, with some perturbative analysis, suggesting that the ordered phase may present RSB for finite-dimensional systems [24]. That makes the situation in the short-range RFIM rather subtle, e.g., the numerical simulations suffer from thermalization problems, in such a way that conclusive results about the nature of the phase transition become difficult to obtain. For the three-dimensional RFIM, recent Monte Carlo simulations detect a jump in the magnetization but no latent heat for both bimodal [25] and Gaussian [26] distributions, whereas high-temperature series expansions [27] and a zero-temperature scaling analysis [28] find a continuous transition for both distributions. However, in four dimensions the same zero-temperature analysis [28] leads to a first-order phase transition in the bimodal case and a continuous one for a Gaussian distribution, in agreement with the mean-field predictions.

Although they may present common properties, particularly at large random fields, the ISG and RFIM have been treated, most of the time, separately; a few works have considered the two systems together [29,30]. However, many systems in nature are properly described through a spin glass in the presence of a random magnetic field. As examples one may mention the proton and deuteron glasses [30], which are mixtures of hydrogen-bonded ferroelectric and antiferroelectrics, considered as the electric counterparts of spin glasses. On the other hand, many diluted antiferromagnets, whose prototype is $Fe_xZn_{1-x}F_2$, due to large crystal-field anisotropies, when submitted to an external uniform magnetic field become good experimental realizations of the RFIM [31] for large enough values of the concentration x; as x decreases, they behave like Ising spin glasses. In the $Fe_xZn_{1-x}F_2$ case, for $x \geqslant 0.40$ one gets a RFIM, whereas for $x \leqslant 0.24$ it becomes an ISG. However, for intermediate concentrations $(0.24 \leqslant x \leqslant 0.40)$ one may have both behaviors [RFIM (ISG) for small (large) magnetic fields], with a crossover between them; that is clearly observed in measurements of $Fe_{0.31}Zn_{0.69}F_2$ [32]. Obviously, such systems are expected to be properly described through a model that is capable of presenting both spin-glass and random-field characteristics; the SK model under a Gaussian random field [29] was able to present the crossover observed in $Fe_{0.31}Zn_{0.69}F_2$.

In this paper we study the SK model in the presence of a bimodal random field; we show that this system presents a first-order phase transition, depending on the magnitude of the random field. This model should be relevant for the description of diluted antiferromagnets exhibiting first-order phase transitions such as $Fe_xMg_{1-x}Cl_2$ [31]. In the next section we define the model and, using the replica method, find its free-energy density and equations of state. In Sec. III we exhibit and discuss the phase diagrams. Finally, in Sec. IV we present our conclusions.

II. THE MODEL AND REPLICA FORMALISM

The SK model in the presence of an external random magnetic field is defined in terms of the Hamiltonian [29]

$$\mathcal{H} = -\sum_{(i,j)} J_{ij} S_i S_j - \sum_i h_i S_i, \tag{2.1}$$

where $S_i = \pm 1$, with $i = 1,2,\ldots,N$, and the interactions are infinite-range-like, i.e., the sum $\Sigma_{i,j}$ applies to all distinct pairs of spins. The coupling constants $\{J_{ij}\}$ and the random fields $\{h_i\}$ are quenched variables, following independent probability distributions

$$P(J_{ij}) = \left(\frac{N}{2\pi J^2}\right)^{1/2} \exp\left[-\frac{N}{2J^2}\left(J_{ij} - \frac{J_0}{N}\right)^2\right], \tag{2.2}$$

$$P(h_i) = p\,\delta(h_i - h_0) + (1-p)\,\delta(h_i + h_0). \tag{2.3}$$

For a given realization of bonds and site fields $(\{J_{ij}\},\{h_i\})$, one has a corresponding free energy $F(\{J_{ij}\},\{h_i\})$ such that the average over the disorder $[\ ]_{J,h}$ may be performed as independent integrals

$$[F(\{J_{ij}\},\{h_i\})]_{J,h} = \int \prod_{(i,j)} [dJ_{ij} P(J_{ij})] \prod_i [dh_i P(h_i)] F(\{J_{ij}\},\{h_i\}). \tag{2.4}$$

The usual procedure consists in applying the replica method [2] in order to get the free energy per spin as

$$-\beta f = \lim_{N\to\infty} \frac{1}{N} [\ln Z(\{J_{ij}\},\{h_i\})]_{J,h} = \lim_{N\to\infty} \lim_{n\to 0} \frac{1}{Nn} ([Z^n]_{J,h} - 1), \tag{2.5}$$

where Z^n is the partition function of n copies of the system defined in Eq. (2.1) and $\beta = 1/T$ (we work in units $k_B = 1$). Standard calculations lead to

$$\beta f = -\frac{(\beta J)^2}{4} + \lim_{n\to 0} \frac{1}{n} \min g(m^\alpha, q^{\alpha\beta}), \tag{2.6}$$

where

$$g(m^\alpha, q^{\alpha\beta}) = \frac{\beta J_0}{2} \sum_\alpha (m^\alpha)^2 + \frac{(\beta J)^2}{2} \sum_{(\alpha,\beta)} (q^{\alpha\beta})^2 - p \ln \mathrm{Tr}_\alpha \exp(\mathcal{H}_{\mathrm{eff}}^{+}) - (1-p) \ln \mathrm{Tr}_\alpha \exp(\mathcal{H}_{\mathrm{eff}}^{-}), \tag{2.7a}$$

$$\mathcal{H}_{\mathrm{eff}}^{\pm} = \beta J_0 \sum_\alpha m^\alpha S^\alpha + (\beta J)^2 \sum_{(\alpha,\beta)} q^{\alpha\beta} S^\alpha S^\beta \pm \beta h_0 \sum_\alpha S^\alpha. \tag{2.7b}$$

In the equations above, the sum indices α and β ($\alpha,\beta = 1,2,\ldots,n$) are replica labels and $\Sigma_{(\alpha,\beta)}$ denote sums over distinct pairs of replicas.

The extrema of the functional $g(m^{\alpha},q^{\alpha\beta})$ give us the equilibrium equations for the magnetization and spin-glass order parameters, respectively,

$$m^{\alpha}=p\langle S^{\alpha}\rangle_{+}+(1-p)\langle S^{\alpha}\rangle_{-}, \tag{2.8a}$$

$$q^{\alpha\beta}=p\langle S^{\alpha}S^{\beta}\rangle_{+}+(1-p)\langle S^{\alpha}S^{\beta}\rangle_{-}, \quad (\alpha\neq\beta), \tag{2.8b}$$

where $\langle\ \rangle_{\pm}$ refer to thermal averages with respect to the ''effective Hamiltonians'' $\mathcal{H}_{\text{eff}}^{\pm}$ in Eq. (2.7b).

If one assumes the replica-symmetry (RS) ansatz [5]

$$m^{\alpha}=m \quad \forall\alpha,$$

$$q^{\alpha\beta}=q \quad \forall(\alpha\beta), \tag{2.9}$$

the free energy per spin [Eq. (2.6)] and the equilibrium conditions [Eqs. (2.8)] become

$$\beta f=-\frac{(\beta J)^2}{4}(1-q)^2+\frac{\beta J_0}{2}m^2, \\ -p\int \mathcal{D}z\ \ln(2\cosh\xi^{+}) \\ -(1-p)\int \mathcal{D}z\ \ln(2\cosh\xi^{-}), \tag{2.10}$$

$$m=p\int \mathcal{D}z\ \tanh\xi^{+}+(1-p)\int \mathcal{D}z\ \tanh\xi^{-}, \tag{2.11}$$

$$q=p\int \mathcal{D}z\ \tanh^2\xi^{+}+(1-p)\int \mathcal{D}z\ \tanh^2\xi^{-}, \tag{2.12}$$

where

$$\int \mathcal{D}z\cdots=\int_{-\infty}^{\infty}\left(\frac{1}{2\pi}\right)^{1/2}dz\ \exp(-z^2/2)\cdots \tag{2.13}$$

and

$$\xi^{\pm}=\beta J_0 m+\beta J q^{1/2}z\pm\beta h_0. \tag{2.14}$$

Although the spin-glass order parameter [Eq. (2.12)] is always induced by the random field, it may still contribute to a nontrivial behavior. The RS solution [Eq. (2.9)] becomes unstable below the Almeida-Thouless [6] line,

$$\left(\frac{T}{J}\right)^2=p\int \mathcal{D}z\ \text{sech}^4\xi^{+}+(1-p)\int \mathcal{D}z\ \text{sech}^4\xi^{-}. \tag{2.15}$$

If $J_0=0$ the integrals involving ξ^{-} in (2.12) and (2.15) may be easily transformed through the change of variables $z\to -z$ in such a way that the AT line is obtained by solving the equations

$$\left(\frac{T}{J}\right)^2=\int \mathcal{D}z\ \text{sech}^4(\beta J q^{1/2}z+\beta h_0), \tag{2.16a}$$

$$q=\int \mathcal{D}z\ \tanh^2(\beta J q^{1/2}z+\beta h_0), \tag{2.16b}$$

which are identical to those of the SK model in the presence of a uniform magnetic field [6]. Therefore, the AT line in the plane magnetic field versus temperature is independent of p and trivially analogous to the one for $p=1$; such a line is invariant under reversal of the field. Hence, in the present problem, $J_0=0$ does not lead to any novel behavior.

For $J_0\neq 0$ there is an AT line for any value of p, given by the solution of Eqs. (2.11), (2.12), and (2.15). At low temperatures $J_0\gg J$ and $J_0\gg h_0$, this line is given by

$$\frac{T}{J}\cong\frac{4}{3}\frac{1}{\sqrt{2\pi}}\left\{p\ \exp\left[-\frac{(J_0+h_0)^2}{2J^2}\right] \\ +(1-p)\exp\left[-\frac{(J_0-h_0)^2}{2J^2}\right]\right\}. \tag{2.17}$$

In the next section we shall consider the phase diagrams for $p=\frac{1}{2}$, $J_0\geqslant 0$, and different values of h_0.

III. PHASE DIAGRAMS

As far as replica symmetry is concerned, the cases $p\neq\frac{1}{2}$ are trivial since both magnetization and spin-glass parameters are nonzero; let us restrict ourselves now to $p=\frac{1}{2}$. In this case the random field induces the parameter q in such a way that there is no spontaneous spin-glass order, like the one found in the SK model, whereas one may still have a phase transition associated with the magnetization. Therefore, analogous to the RFIM, two phases are possible, namely, the ferromagnetic ($m\neq 0$, $q\neq 0$) and the independent ($m=0$, $q\neq 0$) ones. In the RFIM this latter phase is usually denominated paramagnetic; for the present problem, within the RS approximation, we shall keep the nomenclature independent, for reasons that will become clear soon.

The critical frontier separating these two phases may be found by solving the equilibrium equations (2.11) and (2.12); in the case of first-order phase transitions, we shall make use of the free-energy per spin [Eq. (2.10)] as well. Let us then expand Eq. (2.11) in powers of m,

$$m=A_1(q)m+A_3(q)m^3+A_5(q)m^5+O(m^7), \tag{3.1}$$

where the coefficients are given by

$$A_1(q)=\beta J_0[1-\rho_1(q)], \tag{3.2a}$$

$$A_3(q)=-\frac{(\beta J_0)^3}{3}[1-4\rho_1(q)+3\rho_2(q)], \tag{3.2b}$$

$$A_5(q)=\frac{(\beta J_0)^5}{15}[2-17\rho_1(q)+30\rho_2(q)-15\rho_3(q)], \tag{3.2c}$$

with

$$\rho_k(q)=\int \mathcal{D}z\ \tanh^{2k}(\beta J q^{1/2}z+\beta h_0). \tag{3.3}$$

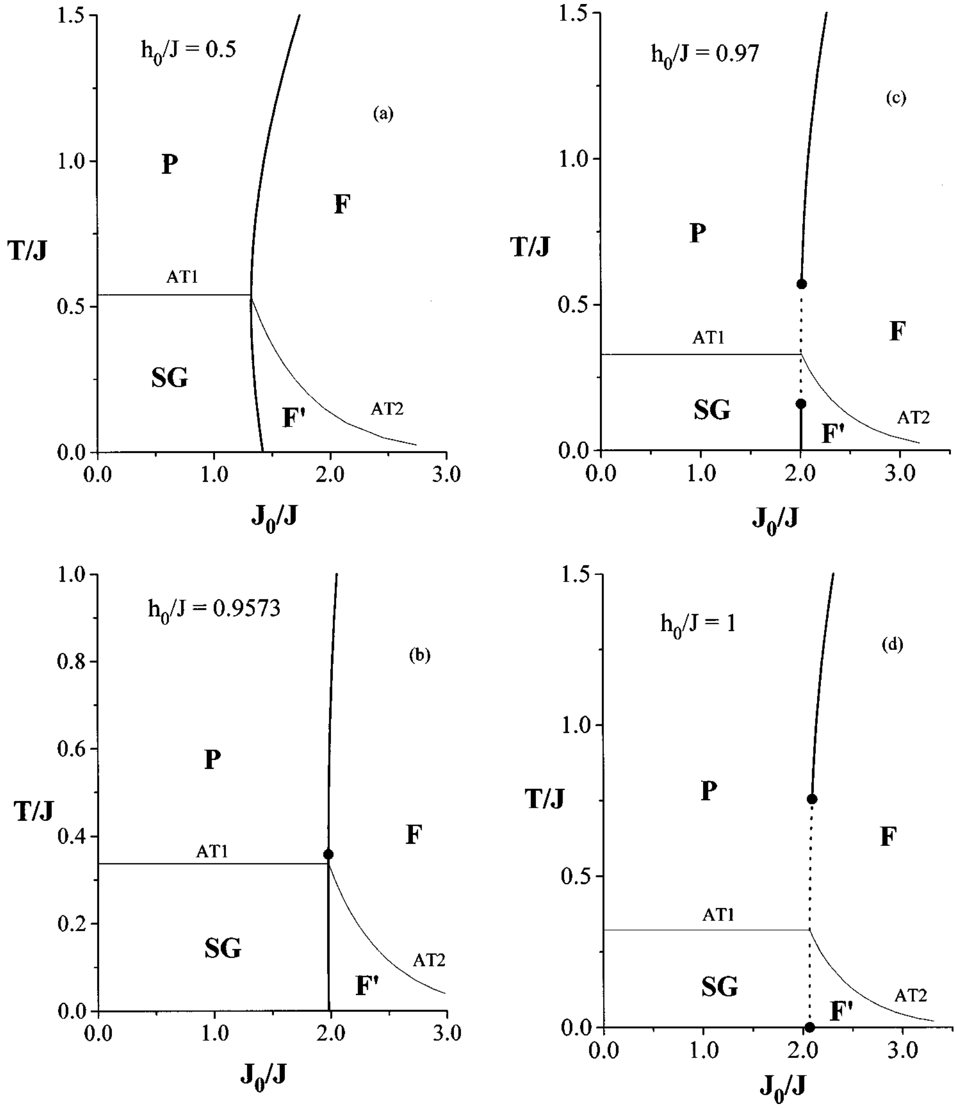

FIG. 1. Typical phase diagrams of the Sherrington-Kirkpatrick model in the presence of a symmetric bimodal random field of magnitude h_0. For the border of the ferromagnetic phase, full lines represent continuous transitions, whereas the dashed ones stand for first-order phase transitions. The black circles along such a border represent points where the coefficient A_3' [see Eq. (3.6b)] is zero. The ferromagnetic critical frontier changes qualitatively for increasing values of h_0: (a) completely continuous; (b) continuous, except for the appearance of a singularity ($A_3'=0$); (c) two tricritical points; and (d) one of the tricritical points collapses with the zero-temperature axis. The lines AT1 and AT2 define the regions of instability of the replica-symmetric solution.

The coefficients in Eqs. (3.2) depend on q [which depends on m through Eq. (2.12)]; expanding Eq. (2.12) in powers of m,

$$q=q_0+\frac{(\beta J_0)^2}{2}\frac{\Gamma}{1-(\beta J)^2\Gamma}m^2+O(m^4), \qquad (3.4)$$

with

$$\Gamma=1-4\rho_1(q_0)+3\rho_2(q_0), \qquad (3.5)$$

where q_0 corresponds to the solution of Eq. (2.12) for $m=0$. Substituting Eq. (3.4) into Eq. (3.1), one gets the m-independent coefficients of the power expansion; we will be particularly interested in the lowest-order ones

$$A_1'=A_1(q_0), \qquad (3.6a)$$

$$A_3'=-\frac{(\beta J_0)^3}{6}\left[\frac{5-(\beta J)^2\Gamma}{1-(\beta J)^2\Gamma}\right]\Gamma. \qquad (3.6b)$$

The critical frontier was determined using standard procedures, as described below.

(i) For continuous phase transitions $A_1'=1$ and $A_3'<0$; a typical case is shown in Fig. 1(a).

(ii) A first-order phase transition occurs whenever $A_1'=1$ and $A_3'>0$; the proper critical frontier was found in this case through a Maxwell construction, i.e., by equating the free energies of the two phases. Typical cases are exhibited in Figs. 1(c), 1(d) and 2 (dashed lines).

(iii) When both types of phase transitions are present, the continuous and first-order critical frontiers meet at a tricritical point [33], which defines the limit of validity of the series expansions; beyond the tricritical point the magnetization is

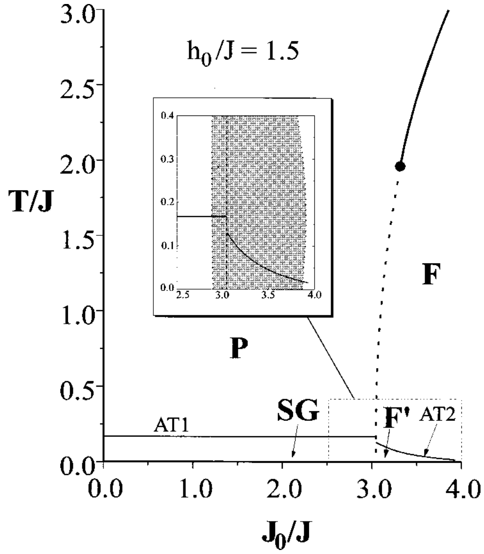

FIG. 2. Phase diagram of the Sherrington-Kirkpatrick model in the presence of a symmetric bimodal random field of magnitude $h_0=1.5J$. The phases and lines follow the same nomenclature used in Fig. 1. The inset is an amplification of the low-temperature rectangular region with $T/J=0.0\rightarrow0.4$ and $J_0/J=2.5\rightarrow4.0$. The gray region in the inset represents the phase coexistence, characteristic of the first-order critical frontier. The line AT1 is valid up to the right end limit of the phase coexistence, whereas AT2 is valid up to the left end limit of this region. Therefore, AT1 and AT2 do not meet at the ferromagnetic critical frontier.

discontinuous. The location of such a point is determined by setting $A_1'=A_3'=0$, with the condition $A_5'<0$ satisfied.

The present problem reveals a curious behavior. For a small range of field magnitudes, the coefficient A_3' changes sign twice: It is negative at high, becomes positive for intermediate, and negative again at low temperatures. In such a case, the critical frontier is composed of two continuous pieces (computed through $A_1'=1$), interpolated by a first-order part (computed by equating the free energies) defining two tricritical points; this occurs for $h_0/J\lesssim1$, as shown in Fig. 1(c).

The finite-temperature phase diagrams of the SK model in the presence of a symmetric bimodal random field are exhibited in Figs. 1 and 2 for increasing values of h_0. One notices that the part of the phase diagram allocated to the ferromagnetic phase gets reduced as h_0 increases. One finds two threshold values of h_0 ($h_0^{(1)}$ and $h_0^{(2)}$), at which the ferromagnetic critical frontier changes qualitatively. For $h_0<h_0^{(1)}$, the frontier is continuous. Two tricritical points are present in the range $h_0^{(1)}\leqslant h_0\leqslant h_0^{(2)}$; these points move in opposite senses in the temperature scale, for increasing values of h_0, in such a way that the lower-temperature one collapses with the zero-temperature axis for $h_0=h_0^{(2)}$ [see Fig. 1(d)]. For $h_0>h_0^{(2)}$ there is a single tricritical point at finite temperatures. We have found numerically that $h_0^{(1)}\approx0.9573J$; at this value, the coefficient A_3' is negative along the whole critical frontier, becoming zero for $T^*\approx0.3582J$ and $J_0^*\approx1.9804J$, corresponding to a singularity (the two tricritical points are superposed), as represented by the black circle in Fig. 1(b). The second threshold value was found analytically, $h_0^{(2)}=J$, through a zero-temperature analysis, which will be discussed next.

We investigated how the above-mentioned critical frontier evolves along the zero-temperature axis; at $T=0$ the spin-glass order parameter is trivial $(q=1)$, whereas the free energy and magnetization become, respectively,

$$f=-\frac{J_0}{2}m^2-\frac{h_0}{2}\left[\mathrm{erf}\left(\frac{J_0m+h_0}{J\sqrt{2}}\right)-\mathrm{erf}\left(\frac{J_0m-h_0}{J\sqrt{2}}\right)\right]-\frac{J}{\sqrt{2\pi}}\left\{\exp\left[-\frac{(J_0m+h_0)^2}{2J^2}\right]+\exp\left[-\frac{(J_0m-h_0)^2}{2J^2}\right]\right\}, \tag{3.7a}$$

$$m=\frac{1}{2}\,\mathrm{erf}\left(\frac{J_0m+h_0}{J\sqrt{2}}\right)+\frac{1}{2}\,\mathrm{erf}\left(\frac{J_0m-h_0}{J\sqrt{2}}\right). \tag{3.7b}$$

Using a similar procedure as the one for finite temperatures, one may expand Eq. (3.7b),

$$m=a_1m+a_3m^3+a_5m^5+O(m^7), \tag{3.8}$$

where

$$a_1=\sqrt{\frac{2}{\pi}}\frac{J_0}{J}\exp\left(-\frac{h_0^2}{2J^2}\right), \tag{3.9a}$$

$$a_3=\frac{1}{6}\sqrt{\frac{2}{\pi}}\left(\frac{J_0}{J}\right)^3\left(\frac{h_0^2}{J^2}-1\right)\exp\left(-\frac{h_0^2}{2J^2}\right), \tag{3.9b}$$

$$a_5=\frac{1}{120}\sqrt{\frac{2}{\pi}}\left(\frac{J_0}{J}\right)^5\left(\frac{h_0^4}{J^4}-6\frac{h_0^2}{J^2}+3\right)\exp\left(-\frac{h_0^2}{2J^2}\right). \tag{3.9c}$$

For $h_0/J<1$ one gets a continuous critical frontier given by

$$\frac{J_0}{J}=\sqrt{\frac{\pi}{2}}\exp\left(\frac{h_0^2}{2J^2}\right), \tag{3.10}$$

which terminates at the tricritical point

$$\frac{h_0}{J}=1,\quad \frac{J_0}{J}=\sqrt{\frac{\pi e}{2}}\approx2.0664. \tag{3.11}$$

This tricritical point corresponds to the zero-temperature collapse shown in Fig. 1(d); one may easily see that the condition $a_5<0$ is satisfied by the coordinates (3.11). Beyond the tricritical point $(h_0/J>1)$, the transition becomes first order; the corresponding critical frontier may be obtained numerically from Eq. (3.7a) by imposing $f(m\neq0)=f(m=0)$, although in the limit $J_0/J,h_0/J\gg1$ one has the analytical result that it should approach the asymptote $J_0=h_0$. The zero-temperature phase diagram is exhibited in Fig. 3.

As mentioned before, the parameter q may still contribute to a nontrivial behavior; this effect is directly related to a stability analysis of the RS solution [6]. Usually two criteria are employed for the identification of a spin-glass phase in infinite-range-interaction models, as we mention below.

(a) Within the RS approximation, the parameter q may become nonzero below a certain temperature, signaling the onset of a spin-glass phase.

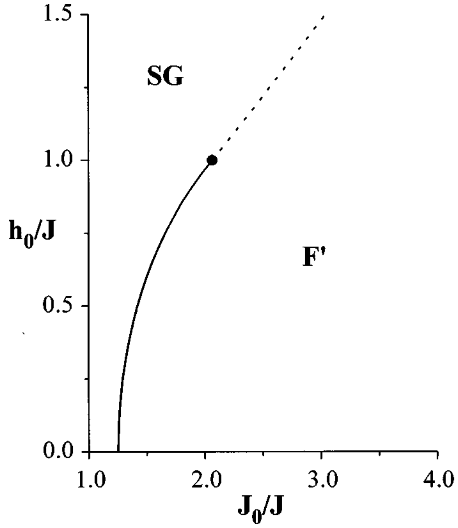

FIG. 3. Zero-temperature phase diagram of the Sherrington-Kirkpatrick model in the presence of a symmetric bimodal random field.

(b) The AT stability analysis normally splits phase diagrams into regions throughout which the RS solution is either stable or unstable. The instability of replica symmetry is usually cured by the introduction of an order-parameter function, a procedure known as replica symmetry breaking [7]. It is very common to associate a spin-glass state with RSB.

Normally, for systems where the RS parameter q becomes nonzero, as mentioned in (a), the AT instability occurs together; in such cases, criteria (a) and (b) coincide in the identification of the spin-glass phase, as happens for the SK model in the absence of a field. However, due to external parameters, a given system may present an induced spin-glass order parameter and an AT-like instability. In this case, the AT line defines two regions in the phase diagram and criterion (b) is employed: In one of them, the spin-glass order parameter is trivially induced and obeys RS (this region is normally denominated a paramagnetic phase); throughout the other one, the spin-glass order parameter is highly nontrivial, defined according to a RSB procedure (this region is usually called a spin-glass phase). As an example of this case, one may mention the SK model in the presence of an external magnetic field.

In the present problem, the AT stability analysis may be carried either to the independent phase ($m=0$) or to the ferromagnetic ($m\neq 0$) one. In the former case, the AT line is given by the solution of Eqs. (2.16) and due to its J_0 independence, one gets horizontal straight lines (AT1), as shown in Figs. 1(a)–1(d) and 2. In the latter, the AT line is obtained by solving Eqs. (2.11), (2.12), and (2.15) for $p=\frac{1}{2}$; in the low-temperature regime one gets the exponential decays of Eq. (2.17), whereas for intermediate temperatures, such equations are solved numerically. The AT lines inside the ferromagnetic region (AT2) are exhibited in Figs. 1(a)–1(d) and 2.

Herein we shall adopt criterion (b) described above for the identification of the paramagnetic and spin-glass phases; in a similar way, the ferromagnetic phase will be split in two parts. The phases exhibited in Figs. 1–3 are identified as paramagnetic (**P**) ($m=0$; q: RS), spin-glass (**SG**) ($m=0$; q: RSB), ferromagnetic (**F**) ($m\neq 0$; q: RS), and mixed ferromagnetic (**F′**) ($m\neq 0$; q: RSB).

For smaller values of h_0 [e.g., Fig. 1(a)], one clearly notices the effect usually denoted ''reentrance:'' By lowering the temperature in the neighborhood of the ferromagnetic border, one comes from a highly disordered phase (**P**) to ordered phases (**F** and **F′**) and then to a less-ordered state (**SG**). This effect is attenuated for increasing values of h_0, similarly to what happens for the SK model in the presence of a Gaussian random field by increasing its distribution width [29].

Below the Almeida-Thouless lines (AT1 and AT2), the RS solution is unstable and a RSB formalism is required; certainly, some changes may occur in a more general type of solution, as we discuss below.

(i) The frontier between the **SG** and **F′** phases is expected to become a vertical straight line (i.e., no reentrance), in analogy to what happens for the SK model, according to the Parisi-Toulouse hypothesis [34]. RSB may eliminate the reentrance effects for small values of h_0 (continuous phase transition); on the other hand, a RSB study of this critical frontier in the case of a first-order phase transition is a difficult task. However, we expect that the shape of the ferromagnetic border for h_0/J greater than or of the order of unity [Figs. 1(b)–1(d) and 2] will not change substantially. For such reasons, the zero-temperature phase diagram exhibited in Fig. 3 will presumably be modified for h_0 small, but its discrepancies should decrease for increasing values of h_0.

(ii) The low-temperature tricritical point for $h_0^{(1)}\leq h_0 \leq h_0^{(2)}$ is, most of the time, inside the unstable region [e.g., Fig. 1(c)]. Whether this tricritical point is an artifact of RS is a question that deserves further investigation.

In the case of continuous phase transitions, the two AT lines (AT1 and AT2) meet at a multicritical point, in the ferromagnetic border, as shown in Figs. 1(a) and 1(b). However, due to the phase-coexistence region in the case of first-order phase transitions, the line AT1 (AT2) goes as far as the right (left) end limit of the phase coexistence, as exhibited in the gray region in the inset of Fig. 2; as a consequence of this, these lines *do not meet* at a point of the ferromagnetic border. Since an AT line signals the instability of the RS solution and does not correspond to a genuine phase transition, we are not aware of any kind of ''Maxwell construction'' that could be used in this case. Therefore, the lines AT1 and AT2 herein exhibited merely represent the solutions of Eqs. (2.11), (2.12), and (2.15) for $p=\frac{1}{2}$.

The RS treatment is appropriate in the region of stability of such solution; hence the border of the ferromagnetic phase for temperatures above the lines AT1 and AT2 will not change under a RSB formalism. In particular, the higher-temperature tricritical point, together with a part of the first-order critical frontier, will persist in more general treatments; this tricritical point is probably reminiscent of the one found in the bimodal RFIM [23].

IV. CONCLUSION

We have studied the Sherrington-Kirkpatrick spin glass in the presence of a bimodal random field, which can assume the values $\pm h_0$ at each site. We have analyzed the phase diagram for the case of a symmetric field distribution, within the replica-symmetry approximation, for which, the spin-glass parameter is always induced by the field, whereas the magnetization becomes nonzero, defining a ferromagnetic phase. By increasing h_0 we have verified that the part of the phase diagram allocated to the ferromagnetic phase decreases; in addition to that, we have found two threshold values ($h_0^{(1)}$ and $h_0^{(2)}$) at which the ferromagnetic critical frontier changes qualitatively: It is completely continuous for $h_0 < h_0^{(1)}$ and presents two tricritical points for $h_0^{(1)} \leq h_0 \leq h_0^{(2)}$ or a single tricritical point for $h_0 > h_0^{(2)}$. By increasing h_0 in the range $h_0^{(1)} \leq h_0 \leq h_0^{(2)}$, we have noticed that the temperatures corresponding to the two tricritical points evolve in opposite senses, i.e., one point moves up, whereas the other one goes down in the temperature scale, in such a way that for $h_0 = h_0^{(2)}$ the lower-temperature tricritical point collapses with the zero-temperature axis.

We have shown that although the spin-glass parameter is always nonzero, it may lead to a nontrivial behavior; the stability analysis of the replica-symmetric solution identifies regions throughout which such a solution becomes unstable. Due to this stability analysis, the phase diagram appears to be composed of four phases: two with zero magnetization [paramagnetic (spin glass)] and two with nonzero magnetization [ferromagnetic (mixed ferromagnetic)], defined in terms of a trivial (nontrivial), i.e., RS (RSB) spin-glass order parameter.

We have found that the higher-temperature tricritical point is always in the region of stability of replica symmetry and should not change under a more general solution; this point is probably reminiscent of the one found for the bimodal RFIM. On the other hand, the lower-temperature tricritical point is, most of the time, inside the unstable region and its existence may be an artifact of the replica-symmetry ansatz.

Due to the first-order phase transition, the limits of stability of the replica-symmetric solution, from either side of the phase-coexistence region, do not meet at the ferromagnetic border, as usually happens for continuous phase transitions.

Which features of the present mean-field picture will predominate in a short-range Ising spin glass in the presence of a bimodal random field turns out to be a question directly related to the survival of mean-field characteristics in the respective short-range versions of the Ising spin glass and random-field model, treated separately. We are not aware of any experimental observations of the results herein reported. However, the diluted antiferromagnet $Fe_xMg_{1-x}Cl_2$ seems to be a good candidate since it has presented evidence of a first-order phase transition [31]; we believe that, for conveniently chosen concentrations, some of the above results may be observed.

ACKNOWLEDGMENTS

We acknowledge Marcia Barbosa for fruitful conversations. We all thank CNPq and CAPES (Brazil) for partial financial support.

[1] For a recent review see V. S. Dotsenko, Phys. Usp. **38**, 475 (1995).

[2] K. Binder and A. P. Young, Rev. Mod. Phys. **58**, 801 (1986); M. Mézard, G. Parisi, and M. A. Virasoro, *Spin Glass Theory and Beyond* (World Scientific, Singapore, 1987); K. H. Fischer and J. A. Hertz, *Spin Glasses* (Cambridge University Press, Cambridge, 1991); D. Chowdhury, *Spin Glasses and Other Frustrated Systems* (World Scientific, Singapore, 1986).

[3] *Spin Glasses and Random Fields*, edited by A. P. Young (World Scientific, Singapore, 1997).

[4] T. Nattermann and J. Villain, Phase Transit. **11**, 5 (1988); T. Nattermann and P. Rujan, Int. J. Mod. Phys. B **3**, 1597 (1989); D. P. Belanger and A. P. Young, J. Magn. Magn. Mater. **100**, 272 (1991).

[5] D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **35**, 1792 (1975).

[6] J. R. L. de Almeida and D. J. Thouless, J. Phys. A **11**, 983 (1978).

[7] G. Parisi, Phys. Rev. Lett. **43**, 1754 (1979); **50**, 1946 (1983).

[8] M. Mézard, G. Parisi, N. Sourlas, G. Toulouse, and M. A. Virasoro, Phys. Rev. Lett. **52**, 1156 (1984); J. Phys. (France) **45**, 843 (1984).

[9] R. N. Bhatt and A. P. Young, Phys. Rev. Lett. **54**, 924 (1985); A. T. Ogielski and I. Morgenstern, *ibid.* **54**, 928 (1985); R. N. Bhatt and A. P. Young, Phys. Rev. B **37**, 5606 (1988); N. Kawashima and A. P. Young, *ibid.* **53**, R484 (1996).

[10] R. Singh and S. Chakravarty, Phys. Rev. Lett. **57**, 245 (1986); L. Klein, J. Adler, A. Aharony, A. B. Harris, and Y. Meir, Phys. Rev. B **43**, 11 249 (1991).

[11] W. L. McMillan, Phys. Rev. B **30**, 476 (1984); **31**, 340 (1985); A. J. Bray and M. A. Moore, J. Phys. C **17**, L463 (1984); Phys. Rev. B **31**, 631 (1985).

[12] A. J. Bray and M. A. Moore, in *Heidelberg Colloquium on Glassy Dynamics*, edited by J. L. van Hemmen and I. Morgenstern, Lecture Notes in Physics Vol. 275 (Springer-Verlag, Heidelberg, 1987).

[13] C. M. Newman and D. L. Stein, Phys. Rev. Lett. **76**, 515 (1996); E. Marinari, G. Parisi, J. Ruiz-Lorenzo, and F. Ritort, *ibid.* **76**, 843 (1996); G. Parisi, e-print cond-mat/9603101; C. M. Newman and D. L. Stein, e-print adap-org/9603001.

[14] D. S. Fisher and D. A. Huse, Phys. Rev. Lett. **56**, 1601 (1986); J. Phys. A **20**, L1005 (1988); Phys. Rev. B **38**, 386 (1988).

[15] W. L. McMillan, J. Phys. C **17**, 3179 (1984).

[16] C. De Dominicis, I. Kondor, and T. Temesvári, in *Spin Glasses and Random Fields* (Ref. [3]).

[17] J. D. Reger, R. N. Bhatt, and A. P. Young, Phys. Rev. Lett. **64**, 1859 (1990); E. R. Grannan and R. E. Hetzel, *ibid.* **67**, 907 (1991); G. Parisi and F. Ritort, J. Phys. A **26**, 6711 (1993); J. C. Ciria, G. Parisi, and F. Ritort, *ibid.* **26**, 6731 (1993); A. Cacciuto, E. Marinari, and G. Parisi, *ibid.* **30**, L263 (1997).

[18] Y. Imry and S. K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[19] S. Fishman and A. Aharony, J. Phys. C **12**, L729 (1979).

[20] J. Cardy Phys. Rev. B **29**, 505 (1984).

[21] J. Imbrie, Phys. Rev. Lett. **53**, 1747 (1984); J. Bricmont and A. Kupiainen, *ibid.* **59**, 1829 (1987).

[22] T. Schneider and E. Pytte, Phys. Rev. B **15**, 1519 (1977).

[23] A. Aharony, Phys. Rev. B **18**, 3318 (1978).

[24] M. Mézard and A. P. Young, Europhys. Lett. **18**, 653 (1992); C. De Dominicis, H. Orland, and T. Temesvári, J. Phys. I **5**, 987 (1995).

[25] H. Rieger and A. P. Young, J. Phys. A **26**, 5279 (1993).

[26] H. Rieger, Phys. Rev. B **52**, 6659 (1995).

[27] M. Gofman, J. Adler, A. Aharony, A. B. Harris, and M. Schwartz, Phys. Rev. B **53**, 6362 (1996).

[28] M. R. Swift, A. J. Bray, A. Maritan, M. Cieplak, and J. R. Banavar, Europhys. Lett. **38**, 273 (1997).

[29] R. F. Soares, F. D. Nobre, and J. R. L. de Almeida, Phys. Rev. B **50**, 6151 (1994).

[30] R. Pirc, B. Tadić, and R. Blinc, Z. Phys. B **61**, 69 (1985); R. Pirc, B. Tadić, and R. Blinc, Phys. Rev. B **36**, 8607 (1987); A. Levstik, C. Filipič, Z. Kutnjak, I. Levstik, R. Pirc, B. Tadić, and R. Blinc, Phys. Rev. Lett. **66**, 2368 (1991); R. Pirc, B. Tadić, and R. Blinc, Physica A **185**, 322 (1992); R. Pirc, R. Blinc, and W. Wiotte, Physica B **182**, 137 (1992).

[31] D. Belanger, in *Spin Glasses and Random Fields* (Ref. [3]).

[32] S. M. Rezende, F. C. Montenegro, U. A. Leitão, and M. D. Coutinho-Filho, in *New Trends in Magnetism*, edited by M. D. Coutinho-Filho and S. M. Rezende (World Scientific, Singapore, 1989); V. Jaccarino and A. R. King, in *New Trends in Magnetism*, edited by M. D. Coutinho-Filho and S. M. Rezende (World Scientific, Singapore, 1989); F. C. Montenegro, A. R. King, V. Jaccarino, S.-J. Han, and D. P. Belanger, Phys. Rev. B **44**, 2155 (1991); D. P. Belanger, Wm. E. Murray, Jr., F. C. Montenegro, A. R. King, V. Jaccarino, and R. W. Erwin, *ibid.* **44**, 2161 (1991).

[33] I. D. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, London 1984), Vol. 9.

[34] G. Parisi and G. Toulouse, J. Phys. (France) Lett. **41**, L361 (1980).